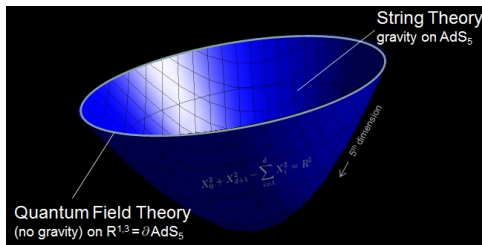


STRONGLY COUPLED QUARK GLUON PLASMA AND THE AdS/CFT CORRESPONDENCE

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- 1 WHAT IS THE QUARK GLUON PLASMA?
- 2 ELLIPTIC FLOW
 - Experimental motivation
 - Strong Coupling and Viscosity
- 3 HYDRODYNAMICS
 - Shear viscosity to entropy density ratio
 - Numerical Results
 - $\mathcal{N} = 4$ Super Yang Mills Theory
- 4 THE ADS/CFT CORRESPONDENCE
- 5 η/s REVISITED
- 6 CONCLUSIONS
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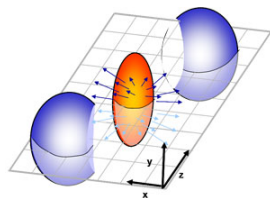
QUARK GLUON PLASMA (QGP)

- Phase of nuclear matter which exists at very high temperature or density.
- Produced in heavy ion collisions ($Au - Au$) at the RHIC (Relativistic Heavy Ion Collider).
- Temperatures at the RHIC are of the order of $4 \times 10^{12} K$.
- QGP is a *strongly coupled* system (sQGP). It is a phase of Quantum Chromodynamics (QCD), the quantum field theory of the strong nuclear force.
- Strong coupling regime of QCD is **non-perturbative**.
- Lattice QCD has been applied, but there are problems with it.
- More recently, string theory-inspired techniques based on the AdS/CFT Correspondence, have been employed.

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ELLIPTIC FLOW



The collision zone between two incoming Au nuclei. Figure courtesy [4].

Elliptic flow: measure of the anisotropy of particle production w.r.t. the reaction plane. Given by

$$v_2 \equiv \left\langle \frac{p_X^2 - p_Y^2}{p_X^2 + p_Y^2} \right\rangle$$

Elliptic Flow can be measured as the second Fourier component in the differential particle yield as a function of the transverse momentum $p_T = \sqrt{p_X^2 + p_Y^2}$:

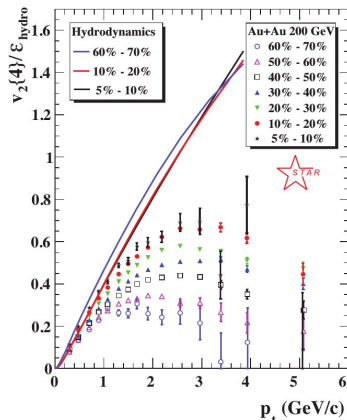
$$\frac{dN}{d\phi} \propto (1 + 2v_2(p_T) \cos(2[\phi - \Psi_{RP}]) + \dots)$$

For a “mid-peripheral collision” ($b \approx 7$ fm), $\langle v_2 \rangle \approx 7\%$. This is large.

$$\frac{\# \text{ particles in X direction}}{\# \text{ particles in Y direction}} = \frac{(1+2v_2)}{(1-2v_2)} \approx 1.3 : 1.$$

$v_2(p_T)$ increases with p_T . At $p_T \sim 1.5$ GeV, $\langle v_2 \rangle \sim 15\%$.

ELLIPTIC FLOW: EXPERIMENTS AND SIMULATIONS



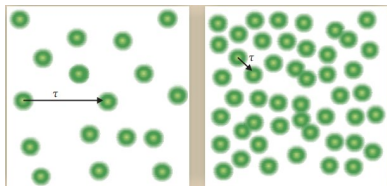
Elliptic flow $v_2(p_T)$ measured by the STAR collaboration. Figure courtesy [1].

- Experiments suggest that the elliptic flow is bounded.
- The ideal gas model (solid curves) overpredicts the flow.

Elliptic flow is an unequivocal signature of the collective (strong coupling) behavior of the Quark Gluon Plasma.

STRONGLY COUPLED FLUIDS AND VISCOSITY

Viscosity is a measure of the tendency to resist flow.



(Left) Weakly interacting fluid, (Right) Strongly interacting fluid.
Figure courtesy [5].

Shear viscosity: measure of how local disturbances in the system are transmitted to the rest of the system through interactions.

Small Mean Free Path: strongly interacting particles (right hand figure).

Small shear viscosities imply **strong coupling**. An ideal gas has no interactions, and hence an infinite shear viscosity.

By dimensional analysis,

$$\eta \sim \rho \bar{v} l_{mfp} \sim \varepsilon \tau_{mfp}$$

where ρ : mass density, \bar{v} : mean velocity, l_{mfp} : mean-free path, ε : energy density, τ_{mfp} : mean-free time.

A “perfect” fluid would have a very low shear viscosity.

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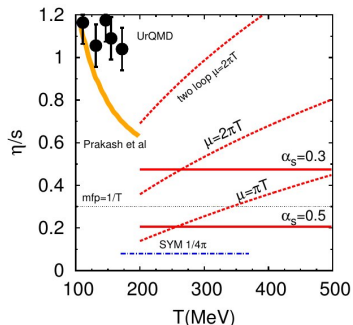
- If $l_{mfp} <$ size of the interaction region, the particles produced are not sensitive to the initial geometry.
- At scales $<$ system size, but $> l_{mfp}$, we expect the system will admit a hydrodynamic description.
- Viscous hydrodynamics yields results that reproduce the flow quite well.

- In a fluid at finite temperature, there is a quantum time scale for energy transport, given by $\tau_{quant} \sim \frac{\hbar}{k_B T}$.
- Particle diffusion coefficient due to finite viscosity (η) estimated by kinetic theory as $\mathcal{D}_n \equiv \frac{\eta}{\varepsilon + \mathcal{P}} \sim v_{th}^2 \tau_R$ (where τ_R : relaxation time scale, v_{th} : the particle velocity, $(\varepsilon + \mathcal{P})$: specific enthalpy).
- Using thermodynamic estimates, $sT \sim \varepsilon v_{th}^2 \sim \mathcal{P} \sim nk_B T$ (n : particle density, \mathcal{P} : pressure, ε : energy density).
- The shear viscosity (η) to entropy density (s) ratio is estimated to be

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B} \frac{\tau_{relaxation}}{\tau_{quantum}}$$

- The quantity η/s is a ratio of the medium relaxation time and the quantum time scale in units of \hbar/k_B .

η/s RATIO: NUMERICAL ESTIMATIONS



Results of numerical calculations of η/s . Figure courtesy [1].

- The η/s ratio has been computed by several groups.
- A common feature is enormous uncertainty around the phase transition temperature $T_C \sim 175$ MeV.
- Is there a strongly coupled theory for which η/s can be computed exactly?
Answer: YES. $\mathcal{N} = 4$ Super Yang Mills (SYM) Theory.

$\mathcal{N} = 4$ SUPER YANG MILLS THEORY

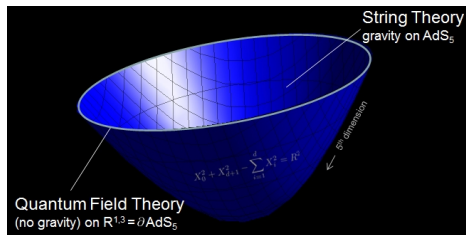
- Supersymmetric Yang Mills Theory, with gauge group $SU(N_c)$ and coupling constant g_{YM} .
- Tunable parameters: N_c and g_{YM} .
- Gauge theory with **1** gauge field, **4** Weyl fermions, and **6** scalars.
- It is a Conformal Field Theory (CFT), i.e. it is invariant under transformations of the group $SO(4, 2)$. Conformal transformations are coordinate transformations that preserve angles.
- Not a superb approximation to “realistic” QCD which has $N_c = 3$ as QCD is **not** supersymmetric, and also **not** conformally invariant.
- But SYM is a useful (and powerful!) **toy model** for theorists.
- Unfortunately, it is still a strongly coupled gauge theory in general, so calculations are difficult.

THE ADS/CFT CORRESPONDENCE

In 1997, Maldacena [6] (and in 1998, Gubser, Klebanov, Polyakov [7], and also Witten [8]) came up with the remarkable conjecture:

THE ADS/CFT CORRESPONDENCE

Type IIB String Theory on an $AdS_5 \times S^5$ background (in $D = 10$) is equivalent to $\mathcal{N} = 4$ SYM Theory on the boundary of the AdS_5 space (in $D = 4$).



- Type IIB String Theory (a gravitational theory) lives in the bulk.
- $\mathcal{N} = 4$ SYM (a non-gravitational gauge theory) lives on the boundary.
- A **gauge-gravity** duality.

THE ADS/CFT CORRESPONDENCE

- Anti de Sitter (AdS) spacetime in 5 dimensions is described (in Poincaré coordinates) by

$$ds^2 = \frac{r^2}{R^2} \underbrace{(-dt^2 + d\mathbf{x}^2)}_{\text{usual } \mathbb{R}^{1,3} \text{ space}} + \frac{R^2}{r^2} dr^2$$

- It is a solution to Einstein's equations in General Relativity with a negative cosmological constant.
- For $r = \text{const.}$, this reduces to $D = 4$ Minkowski space $\mathbb{R}^{1,3}$.
- The AdS/CFT correspondence involves the product of AdS_5 with a sphere S^5 . This is described by:

$$ds^2 = \frac{R^2}{z^2} (-dt^2 + d\mathbf{x}^2 + dz^2) + R^2 d\Omega_5^2$$

where $z = R^2/r$. Here R is the radius of the sphere, as well as of AdS_5 .

THE ADS/CFT CORRESPONDENCE

CFT side ($\mathcal{N} = 4$ SYM)

- Yang-Mills gauge theory
- Gauge group $SU(N_c)$
- Coupling constant: g_{YM}
- $\mathcal{N} = 4$ supersymmetry.
- **1** gauge field, **4** Weyl fermions, **6** scalars.
- Tunable parameters: g_{YM}, N_c .

AdS side (Type IIB String Theory)

- Target space is $AdS_5 \times S^5$.
- Type IIB string theory with finite number of massless fields and infinite number of massive fields.
- Chiral theory with $(2, 0)$ supersymmetry.
- Tunable parameters: R, l_s, g_s .

The coupling constants are related by

$$g_{YM}^2 = 4\pi g_s$$
$$g_{YM}^2 N_c = \frac{R^4}{l_s^4}$$

THE ADS/CFT CORRESPONDENCE

$$g_{YM}^2 = 4\pi g_s$$
$$g_{YM}^2 N_c = \frac{R^4}{l_s^4} = \lambda$$

- Consider $N_c \rightarrow \infty$ and $g_{YM} \rightarrow 0$ as λ is fixed. This means $g_s \rightarrow 0$.
- So classical string theory on $AdS_5 \times S^5$ (no string loops) gives large- N_c dynamics of $\mathcal{N} = 4$ SYM.
- On the other hand, if $R \gg l_s$ and $g_s \ll 1$, then $\lambda \rightarrow \infty$. This corresponds to $g_{YM} \ll 1$ and $N_c \rightarrow \infty$.
- In this regime, wavelengths $\gg l_s$; massive modes decouple. Type IIB string theory reduces to Type IIB supergravity (which is a well defined field theory).

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So calculations in *classical* supergravity tell us about *quantum* field theory.

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THE η/s RATIO REVISITED

- In 2001, Policastro, Son and Starinets [9] computed η/s for $\mathcal{N} = 4$ SYM using the AdS/CFT correspondence.
- Using hydrodynamics, stress energy tensor $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + pg^{\mu\nu}$, where u^μ is a local velocity field.
- To first order in a gradient expansion, $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + pg^{\mu\nu} - \sigma^{\mu\nu}$ where

$$\sigma_{ij} = \eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u^k \right) - \zeta \delta_{ij} \partial_k u^k$$

Conformal invariance enforces $\sigma^{\mu\nu}$ is traceless, so the bulk viscosity $\zeta = 0$.
General covariance modifies σ to:

$$\sigma^{\mu\nu} = \eta P^{\mu\lambda} P^{\nu\rho} \left(\nabla_\lambda u_\rho + \nabla_\rho u_\lambda - \frac{2}{3} g_{\lambda\rho} \nabla \cdot \mathbf{u} \right)$$

where $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$.

THE η/s RATIO REVISITED

- Compute $T^{\mu\nu}$ from the gravity side using AdS/CFT, compare it to hydro form, and extract the pressure density p and the shear viscosity η :

$$p = \frac{\pi^2 N_c^2 T^4}{8}, \quad \eta = \frac{\pi N_c^2 T^3}{8}$$

- The entropy density is $s = \frac{dp}{dT} = \frac{\pi^2 N_c^2 T^3}{2}$. Therefore (for $\hbar = k_B = 1$),

$$\boxed{\frac{\eta}{s} \Big|_{\mathcal{N}=4 SYM} = \frac{1}{4\pi}} \quad (1)$$

- The original calculation suggested positive corrections at 2nd order in the gradient expansion.
- Hence, more generally,

$$\boxed{\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}} \quad (2)$$

in all systems at finite T and μ .

CONCLUSIONS AND CURRENT STATUS

- The AdS/CFT correspondence is currently the only reliable method for tackling strong coupling.
- The bound $\frac{\eta}{s} = \frac{1}{4\pi}$ has been further refined, and is no longer believed to be a rigid lower bound. Effects of $1/N_c$ corrections and higher curvature corrections spoil the bound.
- Precise tests of the AdS/CFT correspondence will require more robust estimation of transport coefficients.
- But this calculation does strengthen the viewpoint that string theory should be treated more as a set of effective field theory *tools* for solving strongly coupled systems.

THANK YOU

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- [10] Title page picture is from <http://www.desy.de/~ikirsch/>.

EXTRA SLIDES: VISCOSITIES FOR DIFFERENT FLUIDS

fluid	P [Pa]	T [K]	η [Pa·s]	η/n [\hbar]	η/s [\hbar/k_B]
H ₂ O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	85	8.2
⁴ He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	0.5	1.9
H ₂ O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	32	2.0
⁴ He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	1.7	0.7
⁶ Li ($a = \infty$)	$12 \cdot 10^{-9}$	$23 \cdot 10^{-6}$	$\leq 1.7 \cdot 10^{-15}$	≤ 1	≤ 0.5
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\leq 5 \cdot 10^{11}$		≤ 0.4

Viscosity η , viscosity over density, and viscosity over entropy density ratios for several fluids. Figure courtesy [2].

EXTRA SLIDES: QUANTUM CHROMODYNAMICS (QCD)

- QCD is the quantum field theory describing strong interactions.
- It is a particular kind of Yang-Mills Theory, with gauge group $SU(3)$.
- A quark belongs to the fundamental representation of $SU(3)$, so it has 3 degrees of freedom called **color**.
- Strong interactions are mediated by gluons, which belong to the adjoint representation of $SU(3)$. So, there are 8 ($= 3^2 - 1$) gluons.
- Unlike QED (which has gauge group $U(1)$, which is Abelian), QCD is **non-Abelian**. Multi gluon interactions are possible (in contrast with QED, which has no multi-photon vertices).
- QCD has two remarkable properties:
 - ★ Asymptotic freedom: the coupling constant reduces at large energies. So, QCD at high energies is perturbative.
 - ★ Confinement: only color-singlet states are observed. Free quarks are not observed.

EXTRA SLIDES: THE ADS/CFT CORRESPONDENCE

CFT side ($\mathcal{N} = 4$ SYM)

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AdS side (Type IIB String Theory)

- Target space is $AdS_5 \times S^5$.
- Type IIB string theory with finite number of massless fields
 - ★ C_0, C_2, C_4 : tensor fields
 - ★ $g_{\mu\nu}, B_{\mu\nu}, \Phi$: graviton, axion, dilaton
 - ★ $\Psi_{\mu,\alpha}^{(1)}, \lambda_{\alpha}^{(1)}$: gravitino + dilatino
 - ★ $\Psi_{\mu,\alpha}^{(2)}, \lambda_{\alpha}^{(2)}$: gravitino + dilatinoand infinite number of massive fields.
- Chiral theory with $(2,0)$ supersymmetry.
- Tunable parameters: R, l_s, g_s .

The coupling constants are related by

$$g_{YM}^2 = 4\pi g_s$$
$$g_{YM}^2 N_c = \frac{R^4}{l_s^4}$$

EXTRA SLIDES: THE η/s RATIO REVISITED

- The stress tensor of the CFT, evaluated at a hypersurface $z = z_0$ is

$$T^\mu{}_\nu = - \lim_{z_0 \rightarrow 0} \frac{\sqrt{-\gamma}}{8\pi G_N} \left[K^\mu{}_\nu - K \delta^\mu{}_\nu + \frac{3}{R} \delta^\mu{}_\nu + \frac{R\mathcal{R}}{4} \delta^\mu{}_\nu - \frac{1}{2} \mathcal{R}^\mu{}_\nu \right]$$

$\gamma_{\mu\nu}$: induced metric on the $z = \epsilon$ slice

$K_{AB} = \nabla_{(A} n_{B)}$, $K = K_{AB} g^{AB}$: extrinsic curvature

n_A : unit normal vector to the constant $z = z_0$ hypersurface

- We want to vary the boundary metric, and extract the viscosity term.
- We use the line element $ds^2 = \frac{R^2}{z^2} \left(-f(z) dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)} \right) + 2\tilde{g}_{xy} \frac{dx dy}{z^2}$ where $f(z) = 1 - \left(\frac{z}{z_h}\right)^4$ and $\tilde{g}_{xy} = e^{-i\omega t} \phi(z)$.
- We impose $\phi(z) \ll 1$ and examine the first order terms in $\phi(z)$ in Einstein's equations $R_{AB} = -\frac{4g_{AB}}{R^2}$. This gives $\square \tilde{g}_{xy} = 0$, the equation of a massless scalar in AdS.
- Boundary conditions $\phi \sim (z - z_h)^\alpha$ for $z \approx z_h$. This gives $\alpha = \pm i\omega z_h/4$. For causal propagation, choose the minus sign. So $\phi(z) = f(z) e^{-i\omega z_h/4}$ which satisfies $\square \tilde{g}_{xy} = 0$ up to $O(\omega^2)$.

EXTRA SLIDES: THE η/s RATIO REVISITED

- Comparing this to the hydrodynamic form of the stress tensor, we get

$$p = \frac{R^3}{16\pi G_N} \frac{1}{z_h^4}, \quad \eta = \frac{R^3}{16\pi G_N} \frac{1}{z_h^3}$$

- Using $z_h = 1/(\pi T)$ and $G_N = \frac{\pi R^3}{2N_c^2}$, we get

$$p = \frac{\pi^2 N_c^2 T^4}{8}, \quad \eta = \frac{\pi N_c^2 T^3}{8}$$

EXTRA SLIDES: AdS/BLACK HOLES

